

# A Rational Approach to Cryptographic Protocols

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## Abstract

This work initiates an analysis of several cryptographic protocols from a rational point of view using a game-theoretical approach, which allows us to represent not only the protocols but also possible misbehaviours of parties. Concretely, several concepts of two-person games and of two-party cryptographic protocols are here combined in order to model the latters as the formers. One of the main advantages of analysing a cryptographic protocol in the game-theory setting is the possibility of describing improved and stronger cryptographic solutions because possible adversarial behaviours may be taken into account directly. With those tools, protocols can be studied in a malicious model in order to find equilibrium conditions that make possible to protect honest parties against all possible strategies of adversaries.

Keywords: Cryptography, Game theory, Protocols verification

## 1 Introduction

The verification of cryptographic protocols has become a subject of great importance with the development of communications and transactions on public channels like Internet. Since Cryptology may be seen as a continuous struggle between cryptographers and cryptanalysts, and Game Theory may be defined as the study of decision making in difficult situations, both fields seem to have certain common scenarios, so it is natural that tools from one area may be applied in the other. In fact, the main objective of this work is to model several two-party cryptographic protocols as two-person games in order to introduce the human factor in the analysis of cryptographic protocols so that it might be

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helpful to solve many security problems which are hard to deal with traditional security primitives.

One of the first approaches that analyses the relationship between cryptographic protocols and games may be found in [1], where an application of game theoretic techniques to the analysis of some multiparty cryptographic protocols for secret exchange was provided. Later, a solution to the problem of determining the existence of two-person games whose payoffs are comparable to those obtained when a Third Trusted Party intervenes was proposed in [2]. Another two recent applications of modern cryptography to game theory were presented respectively in [3], where it was proved that every correlated equilibrium of an original infinitely repeated game can be implemented through public communication only, and in [4], where cryptographic primitives were used to provide correctness and privacy in distributed mechanisms.

Several cryptographic proofs of protocols correctness based on basic fairness were provided in [5], whereas in [6] various formal definitions of different versions of fairness were given. The idea of using game theory as a formal tool to model specific cryptographic protocols such as Fair and Safe Exchange, and Contract Signing was explored in the recent works [7], [8] and [9]. The concept of rational exchange in terms of Nash equilibrium was defined in [10], where it was proved that fair exchange implies rational exchange but not the reverse. Another remarkable reference, [11], described a formal security model for fair signature exchange in terms of games where fairness was defined in a probabilistic way.

Finally, the work [12] should be singled out as the main starting point of this work since there the concept of rationality applied to exchange was introduced. Such a reference also showed the close relationship between the rationality concept and the stimulation for cooperation in ad-hoc networks.

This paper represents a preliminary step of a game-based analysis of general scenarios and different types of two-party cryptographic protocols. Concretely, here the modelling of incentives in the games and desirable conditions of the protocols are described. The structure of the present work is as follows. Section 2 introduces briefly notations and definitions of several game theoretic notions that are used throughout the paper. Then Section 3 provides a basic background on two-party cryptographic protocols. In Sections 4 and 5 a theoretic game model is used to describe and analyse respectively symmetric and asymmetric two-party protocols. Finally, conclusions of the work and comments on further investigation are drawn in Section 6.

## 2 Notations and Definitions

If a group  $P$  of parties or players  $i$  agree to obey certain rules and to act individually or in coalition, the results of their joint action lead to certain situations called outcomes. In such conditions, a game  $G$  defines the set of rules that specify a sequence of actions  $a \in Q$  allowed to the parties.

Concretely, the rules of the game specify what amount of information about all the previous actions and the alternatives that have been chosen can be given to each party before making an specific choice. The game also specifies a termination when some specific sequences of choices are made and no more actions are allowed. Each termination produces an outcome in the form of scores or incomes  $y^+$ , and payments or expenses  $y^-$  for each party. It is assumed that each party  $i$  has a preference relation  $\leq_i$  over the outcomes reflected in his/her scores and payments.

A finite action sequence  $q$  is said to be terminal if it is infinite or if there is no action  $a$  such that  $q$  is followed by  $a$ . The set  $Z$  of terminal action sequences represents all the possible outcomes of the game. The real-valued function  $y(q) = (y_i(q))_{i \in P}$  that assigns the payoffs for every party  $i$  after every terminal action sequence  $q \in Z$  is called outcome or payoff function. These payoff values may be negative, in which case they are interpreted as losses. Also these payoffs may verify that  $\sum_{i \in P} y_i(q) = 0$  for any  $q \in Z$ , in which case the game is called zero-sum.

The preference relations of the parties are often represented in terms of their payoffs in such a way that for any  $q, q' \in Z$  and  $i \in P$ ,  $q \leq_i q'$  iff  $y_i(q) \leq y_i(q')$ . On the other hand, the so called utility function  $u_i$  is just a mathematical representation of  $i$ 's preferences.

A strategy of party  $i \in P$  is a function  $s_i \in S_i$  that assigns an action which is available after  $q$  for party  $i$ , to every non-terminal action sequence  $q \in Q \setminus Z$  such that  $i$  is the following party in choosing an action after  $q$ . A strategy profile is a vector  $(s_i)_{i \in P}$  of strategies, where each  $s_i$  is a member of  $S_i$ . The notation  $(s_j, (s_i)_{i \in P \setminus j})$  is used to emphasise that the strategy profile specifies strategy  $s_j$  for party  $j$ . Finally, let  $o((s_i)_{i \in P})$  denote the resulting outcome when the parties follow the strategies in the strategy profile  $(s_i)_{i \in P}$ .

A strategy profile  $(s^*_i)_{i \in P}$  is called a Nash equilibrium iff for every party  $j \in P$  we have that  $o(s_j, (s^*_i)_{i \in P \setminus j}) \leq_j o(s^*_j, (s^*_i)_{i \in P \setminus j})$ . This means that if every party  $i$  other than  $j$  follows strategy  $s^*_i$ , then party  $j$  is also motivated to follow strategy  $s^*_j$ . So, in Nash equilibrium the choices depend on the other's possible strategies.

### 3 Cryptographic Protocols Concepts

A two-party cryptographic protocol may be defined as the specification of an agreed set of rules on the computations and communications that need to be performed by two entities,  $A$  (Alice) and  $B$  (Bob), over a communication network, in order to accomplish some mutually desirable goal, which is usually something more than simple secrecy. Several essential properties of cryptographic protocols are the following:

1. Correctness, which guarantees that every honest party should get his/her agreed output.

2. Privacy, which includes the protection of every party's secrets.
3. Fairness, which means that if a dishonest party exists, then neither he/she may gain anything valuable, nor honest party may lose anything valuable.

In the game-theoretic model two new properties regarding dishonest behaviours can be defined

1. Exclusiveness, which implies that one or both parties cannot receive their agreed output.
2. Voyeurism, which is the contrary of privacy because it implies that one or both parties may discover the other's secret.

Note that the previous definition of fairness agrees with the rationality concept described in [10] because fairness here is a property which is understood more practical than theoretical. In other words, protocols are here defined according to their practical security against any kind of adversaries.

It is assumed that at each step a party receives the message that was sent by the other party at the previous step, performs some private computation and sends some message (possibly none) to the other party. So, a two-party cryptographic protocol may be seen as a repeated game formed by a sequence of iterations of the following two communication phases:

- 1)Send: Party  $A$  ( $B$ ) sends to  $B$  ( $A$ ) a message  $M$  generated depending on her (his) state.
- 2)Receive: Party  $A$  ( $B$ ) receives from  $B$  ( $A$ ) a message  $M$  and makes a state transition.

Thus, we are implicitly assuming that the system is synchronous (parties know the time and must decide what message to send in each round before receiving any message sent to them in that round), communication is guaranteed, and messages take exactly one round to arrive. These assumptions are critical to the correctness of the protocols. Also, for the sake of simplicity, in this paper the non-intentional loss of control over message  $M$  is considered as a delivery, so  $rcv_A(M)$  ( $rcv_B(M)$ ) denotes both the cases when party  $B(A)$  sends message  $M$  to  $A(B)$ , and when  $A(B)$  is able to receive it.

In order to formalise the notion of cryptographic protocols in terms of functions, we denote by  $f$  a two-argument finite function,  $f : X_A \times X_B \rightarrow Y_A \times Y_B$  where  $X_i$  and  $Y_i$ ,  $i \in \{A, B\}$ , represent respectively input and output sets for party  $i$ . Intuitively, a two-party cryptographic protocol may be generally described through a two-variable function  $f$  whose output is defined by the expression  $f(M_A, M_B) = (f_A(M_A, M_B), f_B(M_A, M_B))$ , where it is understood that party  $i$  receives the output of  $f_i$  on inputs  $M_A$  and  $M_B$ .

As aforementioned, two-party cryptographic protocols include a series of message exchanges between both parties over a communication network. So, the possibility always exists that one or both parties will cheat to gain some

advantage or that some external agent will interfere with normal communications. The simplest situation occurs when each party functions asynchronously from the other party and makes inferences by combining a priori knowledge with properties of the received messages, determining information that is not immediately apparent, so such inferences must be taken into account in determining security. In a worst case analysis of a protocol, one must assume that any party may try to subvert the protocol. So, when designing a two-party cryptographic protocol one of two possible models should be considered:

- Semi-honest model: When it is assumed that the protocol is cooperative and both parties follow the protocol properly in such a way that they help each other to compute  $f_i(M_A, M_B)$ , but curious parties may keep a record of all the information received during the execution and use it to make a later attack.
- Malicious model: Where it is assumed that parties may deviate from the protocol. In this case, during the interaction, each party acts non cooperatively and has different choices which may determine the output of the protocol.

We are interested in obtaining guarantees provided by the definition of the protocols when one of both parties misbehaves in an arbitrary way. Consequently, this work is conducted within the malicious model where it is assumed that either  $A$  or  $B$  does not follow the protocol properly. In such a model the security of a cryptographic protocol should refer to its ability to withstand attacks by certain types of cheaters or enemies, in such a way that essential properties such as correctness, privacy and fairness hold despite such possible attacks. So, the main interest of this work will be the description of honest strategy profiles for every analysed protocol such that whenever the strategy of some party is honest, the other party has no incentive to deviate from the protocol, which is closely related to Nash equilibrium conditions.

Apparently, any two-party cryptographic protocols might be best modelled with a zero-sum game because every situation that is dishonestly advantageous for a party should be disadvantageous for the other. In fact this is not the case of many protocols. In general, most two-party cryptographic protocols are represented by non-positive sum games (i.e. games in which the sum of the payoffs of the players is always less than or equal to 0). Those games in which the sum of the payoffs can be positive should be generally discarded because they imply that both parties could agree on behaving dishonestly and receive positive payoffs.

In particular, the payoff  $y_i(q)$  of a party  $i$ , assigned after a terminal action sequence  $q$  may defined as  $y_i(q) = y^+_i(q) - y^-_i(q)$ , where  $y^+_i(q)$  and  $y^-_i(q)$  represent respectively the incomes and expenses of  $i$  after  $q$ . These incomes and expenses functions will be defined in terms of utilities according to the concrete definitions of each protocol. Here the utility that a secret  $M_j$  is worth to party  $i$

is denoted by  $u_{ij} = u_i(M_j)$ , value which may be difficult to quantify in practical situations.

A two-party cryptographic protocol is said to be closed when if a party gains something, then the other party must lose something. This property may be expressed in terms of the incomes and expenses functions in the following way:  $\forall q \in Z, y^+_i(q) > 0 \Rightarrow y^-_j(q) > 0$ . Note that in this work the closeness of the protocols is assumed since in the definition of the payoff function we always consider both the wish of one party to know the other's secret and the wish of the other party to prevent that from happening.

According to the aforementioned functional definition of a two-party cryptographic protocol  $f$ , at the end of the execution, party  $i$  should receive the output of  $f_i$  on secrets  $M_A$  and  $M_B$ . Depending on whether  $f_A = f_B$  we may distinguish between symmetric and asymmetric protocols. From the first group, in the next sections we will study the protocols of Fair Exchange, Secure Two-Party Computation and Coin Flipping. On the other hand, representative protocols of the group of asymmetric protocols are Oblivious Transfer, Bit Commitment and Zero Knowledge Proof. This classification is important for the proposed game theoretic model because it implies the translation to a symmetric game where possible payoffs and outputs of both parties coincide, or to asymmetric games where that does not occur.

In the following sections several symmetric and asymmetric protocols are analysed according to a game-theoretic model. For every analysed protocol we define income, expense and payoff functions for each party in every possible combination of behaviours and misbehaviours of parties, and make rather minimal assumptions about several matters such as the preferences of the parties in order to guarantee the existence of a honest strategy profile being a Nash equilibrium. Although the possibility of misbehaviours by both parties is here considered, in this paper we analyse specially the case when exactly one of them is dishonest. Note that if this assumption is not fulfilled, there might be some dishonest strategy that dominates the corresponding honest strategy, and in such conditions rational parties would be consequently dishonest.

## 4 Symmetric Protocols

### 4.1 Fair Exchange

Fair Exchange is a cryptographic protocol for exchanging secrets  $M_A$  and  $M_B$  between two parties  $A$  and  $B$  so that if  $A$  behaves correctly, then party  $B$  cannot get  $A$ 's secret ( $M_A$ ) unless  $A$  gets  $B$ 's secret ( $M_B$ ), and vice versa. According to this definition, possible descriptions of non-null values of the incomes and expenses functions  $y^+_i$  and  $y^-_i$  are the following:

$$\begin{aligned} y^+_i(q) &= u_{ij} \text{ if } rcv_i(M_j) \\ y^-_i(q) &= u_{ii} \text{ if } rcv_j(M_i). \end{aligned}$$

Note that if no assumptions or preferences of parties are made, rational parties will simply not send their secrets since this strategy weakly dominates sending the secret. However, since the parties' objective in this protocol is to obtain each other's secret, we are only interested in the states of the protocol tree where  $A$  possesses  $B$ 's secret and the ones where  $B$  possesses  $A$ 's secret. So, one property that utility  $u_{ij}$  should verify in order to avoid a possible coalition between two dishonest parties is the following:  $u_{ij} > u_{ii} > 0, \forall i, j \in \{A, B\}$ . For example, such utilities might reflect the interests of both parties to participate cooperatively if the protocol is run correctly. In this way, parties value correctness over privacy, and the payoff  $y_i(q)$  of party  $i$  can take only four possible values:  $-u_{ii} < 0 < u_{ij} - u_{ii} < u_{ij}$  corresponding respectively to the four possible terminal action sequence when  $rcv_j(M_i) \leq_i rcv_i(\emptyset) \wedge rcv_j(\emptyset) \leq_i rcv_i(M_j) \wedge rcv_j(M_i) \leq_i rcv_i(M_j)$ .

Fairness property ensures that if  $i$  is honest, then the other party  $j$  cannot get  $i$ 's secret unless  $i$  gets  $j$ 's secret. So, in terms of incomes and expenses functions we have that if  $i$ 's strategy  $s_i^*$  is honest, then for every strategy of  $j, s_j$ : if  $y_j^+(o(s_i^*, s_j)) = u_{ji} \Rightarrow y_i^+(o(s_i^*, s_j)) = u_{ij}$ .

So, it may be stated that in a rational fair exchange protocol where both parties have incentives to send their secrets, honest strategies are Nash equilibrium because if one party follows a honest strategy, then the other party is also motivated to behave honestly because he/she loses or at least does not gain anything by not doing so.

Examples of fair exchange include Contract Signing and Certified Mail protocols [13]. In the former, both parties  $A$  and  $B$  want to exchange simultaneously signed contracts in such a way that none of them can obtain the signature of the other without having signed the contract and that none of them can repudiate his or her own signature. On the other hand, in Certified Mail  $A$  wants to send a mail  $M_A$  to  $B$  so that  $B$  can read the mail  $M_A$  if and only if  $A$  receives the corresponding return receipt  $M_B$ . Consequently, a conclusion similar to the obtained for fair exchange may be extracted for both cases of contract signing and certified mail protocols.

## 4.2 Secure Two-Party Computation

The general protocol known as Secure Two-Party Computation allows that two parties  $A$  and  $B$  with secret inputs  $M_A$  and  $M_B$  to evaluate a common value  $f_A(M_A, M_B) = f_B(M_A, M_B) = g(M_A, M_B) = g$  in a manner where neither party learns more than necessary. This protocol is the two-party version of the multiparty protocol known as Secure Function Evaluation. There are various definitions and models for Secure Two-Party Computation [14] and indeed the above definition describes just one of them. For example, one might consider an asymmetric version where only  $A$  receives the output. However, this work deals with this symmetric version where both parties learn the value  $g$ .

A possible description of the incomes and expenses functions  $y^+_i$  and  $y^-_i$  that verifies the previous definition is as follows, where  $k > 1$ :

$$y^+_i(q) = \begin{cases} u_{ij} & \text{if } rcv_i(M_j) \\ ku_i(g) & \text{if } rcv_i(g) \\ u_{ij} + ku_i(g) & \text{if } rcv_i(M_j, g) \\ 0 & \text{otherwise} \end{cases}$$

$$y^-_i(q) = \begin{cases} u_{ii} & \text{if } rcv_j(M_i) \\ u_i(g) & \text{if } rcv_j(g) \\ u_{ii} + u_i(g) & \text{if } rcv_j(M_i, g) \\ 0 & \text{otherwise} \end{cases}$$

A serious problem of this protocol arises when there is no way to force a party to use his/her correct input. So, according to privacy property, and in order to avoid a possible coalition between dishonest parties, we there should be assumed that the following inequality holds:  $u_i(g) < u_{ij} < ku_i(g) < u_{ii}, \forall i, j \in A, B$  which implies that: exclusiveness  $\leq_i$  voyeurism  $\leq_i$  correctness  $\leq_i$  privacy.

If the utility of  $g(M_A, M_B)$  is the same for both parties,  $u = u_A(g) = u_B(g)$ , the payoff  $y_i(q)$  of party  $i$  may take the following sixteen possible values:  $-u - u_{ii} < -u_{ii} < u_{ij} - u_{ii} - u < (k-1)u - u_{ii} < ku - u_{ii} < -u, u_{ij} - u_{ii}, (k-1)u + u_{ij} - u_{ii} < 0, ku + u_{ij} - u_{ii} < u_{ij} - u < u_{ij}, (k-1)u < ku < (k-1)u + u_{ij} < ku + u_{ij}$  corresponding respectively to the sixteen possible terminal action sequence when  $rcv_j(g, M_i) \leq_i rcv_j(M_i) \leq_i rcv_j(g, M_i) \wedge rcv_i(M_j) \leq_i rcv_i(g) \wedge rcv_j(g, M_i) \leq_i rcv_j(M_i) \wedge rcv_i(g) \leq_i rcv_j(g), rcv_j(M_i) \wedge rcv_i(M_j), rcv_j(g, M_i) \wedge rcv_i(g, M_j) \leq_i rcv_i(\emptyset) \wedge rcv_j(\emptyset), rcv_i(g, M_j) \wedge rcv_j(M_i) \leq_i rcv_i(M_j) \wedge rcv_j(g) \leq_i rcv_i(M_j), rcv_i(g) \wedge rcv_j(g) \leq_i rcv_i(g) \leq_i rcv_i(g, M_j) \wedge rcv_j(g) \leq_i rcv_i(g, M_j)$ .

A rational secure two-party computation protocol ensures that no party receive the other party's secret and that if party  $i$  is honest, then the other party  $j$  cannot get  $g(M_A, M_B)$  unless  $i$  gets it. So, in terms of incomes and expenses functions we have that if  $i$ 's strategy  $s_i^*$  is honest, then for every strategy of  $j$ ,  $s_j$ : if  $y_j^+(o(s_i^*, s_j)) = ku_j(g) \Rightarrow y_i^+(o(s_i^*, s_j)) = ku_i(g)$ , and if  $y_j^+(o(s_i^*, s_j)) = u_{ji} + ku_j(g) \Rightarrow y_i^+(o(s_i^*, s_j)) = u_{ij} + ku_i(g)$ . So, in rational secure two-party computation protocol, honest strategies hold Nash equilibrium conditions.

### 4.3 Coin Flipping

Coin flipping protocols are used where two parties  $A$  and  $B$  want to generate jointly a common random binary sequence  $M$ . According to this definition, possible descriptions of non-null additive values of the incomes and expenses functions  $y^+_i$  and  $y^-_i$  are the following, where  $k > 1$ :

$$\begin{aligned} y^+_i(q) &= u_i(M) \text{ if } M \text{ is selected by } i \\ y^+_i(q) &= ku_i(M) \text{ if } rcv_i(M) \\ y^-_i(q) &= ku_i(M) \text{ if } M \text{ is selected by } j \\ y^-_i(q) &= u_i(M) \text{ if } rcv_j(M). \end{aligned}$$

In this way, according to preferences of parties, correctness and voyeurism are valued over exclusiveness and privacy, and the payoff  $y_i(q)$  of party  $i$  can take five possible values:  $-ku_i(M) < -u_i(M) < 0 < u_i(M) < ku_i(M)$  corresponding respectively to the five possible terminal action sequence when  $M$  is selected by  $j \leq_i rcv_j(M) \leq_i rcv_j(M) \wedge rcv_i(M) \leq_i M$  is selected by  $i \leq_i rcv_i(M)$

Again fairness property ensures that either both parties get the agreed outcome or neither does, so if party  $i$  is honest, then the other party  $j$  cannot get the randomly generated sequence  $M$  before. So, in terms of incomes and expenses functions we have that if  $i$ 's strategy  $s_i^*$  is honest, then for every strategy of  $j$ ,  $s_j$ , if  $y_j^+(o(s_i^*, s_j)) = ku_j(M) \Rightarrow y_i^+(o(s_i^*, s_j)) = ku_i(M)$ . Consequently, honest strategies in rational coin flipping protocols are Nash equilibrium.

## 5 Asymmetric Protocols

### 5.1 Oblivious Transfer

A major component in the construction of Secure Two-Party Computation protocols is the Oblivious Transfer protocol since it has been proved that a Secure Two-Party Computation can be always built using calls to an Oblivious Transfer protocol [15]. So, the term Oblivious Transfer refers usually to several different versions of asymmetric Secure Two-Party Computation protocols, all of which turned out to be equivalent. However, the definition that will be used in this work is the following. An Oblivious Transfer may be defined as a protocol whose goal is to enable one party  $A$  to transfer a secret to another party  $B$  in such a way that the information is transferred with a probability  $1/2$ , and when concluding the protocol  $B$  knows with absolute certainty whether he has got the secret or not, but  $A$  does not know it.

Possible descriptions of additive incomes and expenses functions  $y_i^+$  and  $y_i^-$  are the following, where  $k > 1$ :

$y_A^-(q) = u_A(M)$  and  $y_B^+(q) = u_B(M)$  if  $rcv_B(M)$   
 $y_A^+(q) = ku_A(M)$  and  $y_B^-(q) = (k+1)u_B(M)$  if  $A$  knows whether  $rcv_B(M)$  or not.

If no assumption is made on  $A$ ' interest to participate in a correct protocol, then there may be a problem because a rational party  $A$  would simply not send her secret. Consequently, the described model implies that party  $A$  should value voyeurism over exclusiveness, whereas party  $B$  should value privacy over correctness. On the one hand, the payoff functions of party  $A$  can take the following four values:  $-u_A(M) < (k-1)u_A(M) < 0 < ku_A(M)$  corresponding respectively to the four possible terminal action sequence when  $rcv_B(M) \leq_i rcv_B(M) \wedge A$  knows it  $\leq_i rcv_B(\emptyset) \leq_i rcv_B(\emptyset) \wedge A$  knows it. On the other hand, the payoff functions of party  $B$  can take the following four values:  $(-k-1)u_B(M) < -ku_B(M) < 0 < u_B(M)$  corresponding respectively to the four possible terminal action sequence when  $rcv_B(\emptyset) \wedge A$  knows it  $\leq_i rcv_B(M) \wedge A$

knows it  $\leq_i rcv_B(\emptyset) \leq_i rcv_B(M)$ .

A rational oblivious transfer ensures that if party  $B$  is honest,  $A$  cannot know whether  $B$  received the secret or not, and if party  $A$  is honest,  $B$  receives the secret with probability  $1/2$ . So, in terms of incomes and expenses functions we have that if  $B$ 's strategy  $s_B^*$  is honest, then for every strategy of  $A$ ,  $s_A$ : if  $y_A^+(o(s_B^*, s_A)) = ku_A(M) \Rightarrow y_B^+(o(s_B^*, s_A)) = u_B(M)$ , so  $s_A$  is not a good strategy for  $A$ . From the above it may be stated that honest strategies in rational oblivious transfer hold Nash equilibrium conditions.

## 5.2 Bit Commitment

The goal pursued by this two party protocol is twofold: first  $A$  transfers information to  $B$  that can not be changed for her (unalterability property) and such information can not be accessed by  $B$  until the end of the protocol is reached (illegibility property). Originally the aforementioned information consists of only one bit.

When defining utility function the possible frauds should be taken into account for both participants. So, in this case  $B$  would obtain the bit before opening the commitment,  $A$  could also modify the content of the original commitment while the protocol's development. The expenses and incomes of each participants are the following where  $k > 1$ :

$$\begin{aligned} y_A^-(q) &= k \cdot u_A(M), y_B^+ = u_B(M), \text{ if } rcv_B(M) \text{ before the opening stage} \\ y_A^+(q) &= u_A(M), y_B^- = k \cdot u_B(M), \text{ if } A \text{ modifies } M \end{aligned}$$

According to the previous values, the payoff for each party has the values  $0, -k \cdot u_i(M), u_i(M)$  and  $(1 - k) \cdot u_i(M), i \in \{A, B\}$ . From these utility functions it can be deduced that the honest behaviour of party  $A$  implies  $B$ 'honesty, since the other possibilities convey non positive payoffs. Again honest strategies have Nash equilibrium associated. Furthermore it can be deduced that party  $A$  associate a bigger weight to privacy property than to exclusiveness. On the other hand,  $B$ 's preferences single out correctness property compared to voyeurism.

## 5.3 Zero-Knowledge Proofs

A zero-knowledge protocol allows party  $A$  to convince  $B$  that she knows some information but without leaking anything about the secret. The two dishonest possibilities considered are: party  $A$  does not know the secret or party  $B$  gets the secret, so the corresponding expenses and incomes are

$$\begin{aligned} y_A^- &= k \cdot u_A(M), y_B^+ = u_B(M), \text{ if } rcv_B(M) \\ y_A^+ &= u_A(M), y_B^- = k \cdot u_B(M) \text{ when } A \text{ does not know the secret} \end{aligned}$$

The payoff deduced from those values are  $0, u_i(M)$  and  $-k \cdot u_i(M), i \in \{A, B\}$ . Hence, Nash's equilibrium forces both participants to be honest. According to the previous model, party  $A$  should value privacy over exclusiveness while for party  $B$ , correctness outweighs voyeurism.

## 6 Conclusions

This paper addresses an emergent issue in security: the synergy between security protocols and game theory mechanisms. In particular, the study of several two-party protocols in a game-theoretic model is here initiated, by giving formal definitions of payoffs for each party and ranking properties of exclusiveness, voyeurism, correctness and privacy. This work deals with the idea of modelling cryptographic protocols design as the search of an equilibrium in order to defend honest parties against all possible strategies of malicious parties. So, our first objective has been to illustrate the close connection between protocols and games and to use game theoretic techniques for the definition and analysis of cryptographic protocols so that this model might be used to build more effective and efficient security protocols.

Two subjects that are being object of work in progress are the generalization of the game-theoretic approach followed in this work to multiparty cryptographic protocols, and the analysis of the relationship between properties like fairness and different game theoretic concepts, such as dominant strategic equilibrium. Finally, one direction for further investigation involves the study of the possibility of describing two-party protocols as sequential games instead of repeated games, which might be more convenient in many cases.

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